(b) A job has to be processed over two machines M_1 and M_2 in that order. The distribution of inter-arrival time of the jobs at the first machine is as follows :

Time	1	2	3	4
Probability	0.2	0.2	0.2	0.4

The processing times at the two machines are as follows :

Machine M_1		Machine M_2	
Time (minutes)	Probability	Time (minutes)	Probability
1	0.1	4	0.2
2	0.2	5	0.3
3	0.3	6	0.4
4	0.3	7	0.1
5	0.1		

On the basis of 10 simulation runs, find out the average queue length before Machine M_1 and the average queue length before Machine M_2.

Exam. Code : 211004
Subject Code : 4999

M.Sc. (Mathematics) 4th Semester

OPERATIONS RESEARCH–II

Paper—MATH-588

Time Allowed—2 Hours] [Maximum Marks—100

Note :— There are **EIGHT** questions of equal marks. Candidates are required to attempt any **FOUR** questions.

1. (a) Obtain the distribution of waiting time of a customer in the system.

   (b) On an average 96 patients per 24 hours day require the service of an emergency clinic. Also on the average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 minutes and that each minute of decrease in the average time would cost Rs. 10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from 4/3 patients to 1/2 a patient ?

2. (a) Derive the formula for the average number of customers in the system for the model (M/M/1) : (N/FIFO).

(b) Assume that trains are coming in a yard at the rate of 30 per day and suppose that the inter-arrival times follow exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines and one is reserved for shunting purposes), then :

(i) Calculate that the probability that the yard is empty.

(ii) Find the average queue length.

3. (a) What do you mean by birth-death model ? Does (M/M/C) : (∞/FIFO) model represent birth-death model ? If so derive the probability of waiting arrival for this model.

(b) A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that when he starts a production run, he can produce 25,000 bearings per day. The cost of holding a bearing in the stock for one year is 20 paisa and the set up cost of a production run is Rs. 180. How frequently should the production runs be made ?

4. (a) Describe the method of purchase inventory problem with one price break.

(b) Find the optimum order quantity for a product for which the price breaks are :

Quantity	**Unit cost (Rs.)**
$0 \leq Q_1 < 500$	10.00
$500 \leq Q_2 < 750$	9.25
$750 \leq Q_3$	8.75

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering is Rs. 100.

5. (a) Explain the replacement policy when the value of money changes with time.

(b) The data collected in running a machine, the cost of which is Rs. 60,000, are given below :

Year	1	2	3	4	5
Resale value (Rs.)	42000	30000	20400	14400	9650
Cost of Spares (Rs.)	4000	4270	4880	5700	6800
Cost of Labour (Rs.)	14000	16000	18000	21000	25000

Determine the optimum period for replacement of the machine.

6. State and prove Mortality Theorem.

7. (a) Explain in detail the concept of event type simulation.

(b) The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous, the rain distribution is :

Event	No rain	1 cm. rain	2 cm. rain	3 cm. rain	4 cm. rain	5 cm. rain
Probability	0.50	0.25	0.15	0.05	0.03	0.02

If it did not rained on the previous day, the rain distribution is :

Event	No rain	1 cm. rain	2 cm. rain	3 cm. rain
Probability	0.75	0.15	0.06	0.04

Simulate city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random number for simulation :

70, 63, 39, 55, 29, 78, 70, 06, 78, 76

Assume that for the first day of simulation it had not rained the day before.

8. (a) An automobile company manufactures around 150 scooters. The daily production varies from 146 to 154 depending upon the availability of raw materials and other working conditions.

Production per day	146	147	148	149	150
Probability	0.04	0.09	0.12	0.14	0.11

Production per day	151	152	153	154
Probability	0.10	0.20	0.12	0.08

The finished scooters are transported in a specially arranged lorry accommodating 150 scooters. Use the following random numbers :

80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57

and simulate the process to find out :

(i) What will be the average number of scooters waiting in the factory ?

(ii) What will be the average number of empty spaces in the lorry ?